

Fig. 7 Height of Mach stem.

## References

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- <sup>4</sup> Chow, W. L. and Addy, A. L., "Interaction between Primary and Secondary Streams of Supersonic Ejector Systems and Their Performance Characteristics," *AIAA Journal*, Vol. 2, No. 4, April, 1964, p. 686.
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- <sup>6</sup> Traugott, S. C., "An Approximate Solution of the Direct Supersonic Blunt-Body Problem for Arbitrary Axisymmetric Shapes," *Journal of the Aerospace Sciences*, Vol. 27, 1960, pp. 361-370.
- <sup>7</sup> Howlett, L. D., "A Study of Nozzle Flow Problems by the Method of Integral Relations," Ph. D. thesis, Dept. of Mechanical and Industrial Engineering, Univ. of Illinois at Urbana-Champaign, Urbana, Ill.

## Errata

### Errata: "Roots of the Cylindrical Shell Characteristic Equation for Harmonic Circumferential Edge Loading"

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IN the aforementioned paper<sup>1</sup> various versions of the cylindrical shell characteristic equation for harmonic circumferential edge-loading were solved numerically and the roots compared. It has been brought to the author's attention<sup>1</sup> that the coefficients of the complete Flügge equation are incorrect. The required changes are that Eqs. (3b) and (3c) should be rewritten as

Table 1 Values of the roots of the complete Flügge characteristic equation for edge-loaded cylindrical shells

| $n$              | $p_{n1}$  | $q_{n1}$  | $p_{n2}$ | $q_{n2}$ |
|------------------|-----------|-----------|----------|----------|
| 0                | 4.04809   | 4.08501   | ...      | ...      |
| 2                | 4.56863   | 3.63890   | 0.45591  | 0.37916  |
| 4                | 6.13012   | 2.98441   | 2.02105  | 1.03738  |
| 6                | 8.05787   | 2.63703   | 3.94978  | 1.38578  |
| 8                | 10.04291  | 2.44489   | 5.93561  | 1.57873  |
| 10               | 12.03906  | 2.32061   | 7.93266  | 1.70390  |
| 12               | 14.03797  | 2.23082   | 9.93260  | 1.79469  |
| 14               | 16.03758  | 2.16084   | 11.93343 | 1.86580  |
| 16               | 18.03728  | 2.10326   | 13.93454 | 1.92464  |
| 18               | 20.03686  | 2.05395   | 15.93577 | 1.97532  |
| 20               | 22.03626  | 2.01043   | 17.93705 | 2.02032  |
| 22               | 24.03546  | 1.97111   | 19.93838 | 2.06120  |
| 24               | 26.03445  | 1.93492   | 21.93977 | 2.09901  |
| 26               | 28.03322  | 1.90113   | 23.94123 | 2.13448  |
| 28               | 30.03178  | 1.96919   | 25.94280 | 2.16811  |
| 30               | 32.03012  | 1.83870   | 27.94447 | 2.20029  |
| 32               | 34.02824  | 1.80934   | 29.94628 | 2.23129  |
| 34               | 36.02613  | 1.78088   | 31.94824 | 2.26132  |
| 36               | 38.02378  | 1.75310   | 33.95038 | 2.29057  |
| 38               | 40.02119  | 1.72585   | 35.95270 | 2.31915  |
| 40               | 42.01835  | 1.69897   | 37.95522 | 2.34718  |
| 42               | 44.01524  | 1.67233   | 39.95797 | 2.37473  |
| 44               | 46.01184  | 1.64581   | 41.96096 | 2.40187  |
| 46               | 48.00816  | 1.61931   | 43.96421 | 2.42868  |
| 48               | 50.00416  | 1.59272   | 45.96774 | 2.45518  |
| 50               | 51.99984  | 1.56593   | 47.97156 | 2.48143  |
| 52               | 53.99519  | 1.53884   | 49.97570 | 2.50746  |
| 54               | 55.99017  | 1.51134   | 51.98018 | 2.53330  |
| 56               | 57.98479  | 1.48332   | 53.98501 | 2.55899  |
| 58               | 59.97901  | 1.45466   | 55.99021 | 2.58454  |
| 60               | 61.97281  | 1.42524   | 57.99582 | 2.60998  |
| 62               | 63.96619  | 1.39492   | 60.00183 | 2.63534  |
| 64               | 65.95912  | 1.36355   | 62.00829 | 2.66064  |
| 66               | 67.95158  | 1.33097   | 64.01520 | 2.68589  |
| 68               | 69.94354  | 1.29698   | 66.02260 | 2.71113  |
| 70               | 71.93499  | 1.26138   | 68.03050 | 2.73636  |
| 72               | 73.92590  | 1.22392   | 70.03893 | 2.76163  |
| 74               | 75.91626  | 1.18431   | 72.04790 | 2.78694  |
| 76               | 77.90603  | 1.14223   | 74.05745 | 2.81233  |
| 78               | 79.89521  | 1.09724   | 76.06759 | 2.83782  |
| 80               | 81.88376  | 1.04886   | 78.07836 | 2.86344  |
| 82               | 83.87166  | 0.99644   | 80.08976 | 2.88922  |
| 84               | 85.85889  | 0.93915   | 82.10182 | 2.91520  |
| 86               | 87.84544  | 0.87586   | 84.11457 | 2.94142  |
| 88               | 89.83128  | 0.80497   | 86.12802 | 2.96790  |
| 90               | 91.81639  | 0.72403   | 88.14219 | 2.99470  |
| 92               | 93.80076  | 0.62893   | 90.15710 | 3.02186  |
| 94               | 95.78438  | 0.51150   | 92.17275 | 3.04943  |
| 96               | 97.76724  | 0.34948   | 94.18917 | 3.07746  |
| 98 <sup>a</sup>  | 99.59688  | 99.90177  | 96.20635 | 3.10602  |
| 100 <sup>a</sup> | 101.31252 | 102.14876 | 98.22430 | 3.13516  |

<sup>a</sup> The complete Flügge equation yields two pairs of real roots, the magnitudes of which are tabulated as  $p_{n1}$  and  $q_{n1}$ .

$$\alpha_2 = 4 + [(11 - 3\nu)/2]k + 9[(1 - \nu)/2]k^2 \quad (3b)$$

$$\alpha_3 = 6 + 3(2 - \nu)k - \nu^2 k^2 \quad (3c)$$

The roots of the various equations were recalculated for  $R/h$  of 10, 20, and 50 and  $n$  varying from 1 to 300 with the use of a double-precision polynomial root extraction routine DPRBM.<sup>2</sup> The corrected results for the complete Flügge equation for  $n$  varying by steps of 2 from 0 to 100 and  $R/h = 10$  are shown in Table 1. Some additional results for  $R/h = 10$  and  $n = 150, 200, 250, 300$  are given for all three characteristic equations in Table 1a. Figure 1 is corrected as shown.

The corrected results indicate that the roots of the complete and simplified Flügge equations are in good agreement over a much larger range of  $n$  than was previously found. In both cases the pattern of the roots shifts from all complex roots to four real roots and four complex roots for large  $n$ . The value of  $n$  at

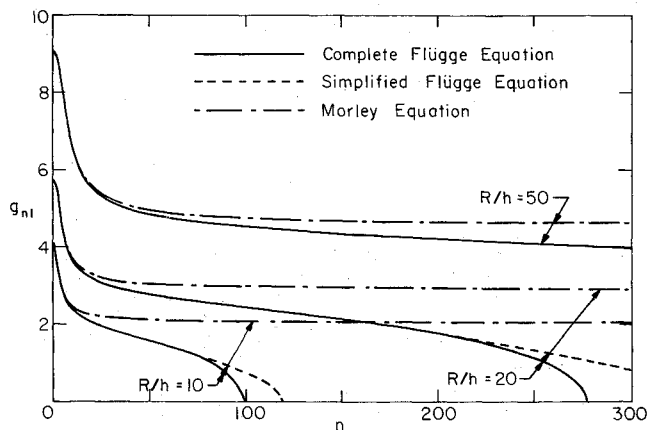


Fig. 1 Comparison of imaginary parts of roots of various characteristic equations.

Table 1a Additional Values of the roots of the various characteristic equations for edge-loaded cylindrical shells

| $n$ | Equation   | $p_{n1}$               | $q_{n1}$               | $p_{n2}$  | $q_{n2}$ |
|-----|------------|------------------------|------------------------|-----------|----------|
| 150 | Complete   | 148.67766 <sup>a</sup> | 153.64343 <sup>a</sup> | 148.77528 | 4.18929  |
|     | Simplified | 151.36940 <sup>a</sup> | 153.16379 <sup>a</sup> | 147.73066 | 3.25097  |
|     | Morley     | 152.03074              | 2.05994                | 147.96593 | 2.00486  |
| 200 | Complete   | 197.07033 <sup>a</sup> | 204.62290 <sup>a</sup> | 199.06941 | 5.60617  |
|     | Simplified | 200.98887 <sup>a</sup> | 203.75899 <sup>a</sup> | 197.62401 | 3.56021  |
|     | Morley     | 202.03115              | 2.05295                | 197.96635 | 2.01175  |
| 250 | Complete   | 245.82860 <sup>a</sup> | 255.57224 <sup>a</sup> | 249.19554 | 7.03704  |
|     | Simplified | 250.78380 <sup>a</sup> | 254.17507 <sup>a</sup> | 247.51891 | 3.82807  |
|     | Morley     | 252.03140              | 2.04892                | 247.96661 | 2.01587  |
| 300 | Complete   | 294.70866 <sup>a</sup> | 306.52728 <sup>a</sup> | 299.25778 | 8.46025  |
|     | Simplified | 300.65097 <sup>a</sup> | 304.51040 <sup>a</sup> | 297.41794 | 4.06524  |
|     | Morley     | 302.03156              | 2.04616                | 297.96678 | 2.01862  |

<sup>a</sup> The complete Flügge equation yields two pairs of real roots, the magnitudes of which are tabulated as  $p_{n1}$  and  $q_{n1}$ .

Table 2 Some values of the coefficients of the characteristic equation  $A_0 + A_1\lambda_n^2 + A_2\lambda_n^4 + A_3\lambda_n^6 + A_4\lambda_n^8 = 0$  ( $R/h = 10$ ,  $\nu = 0.30$ )

|     |       |                 |                 |                 |
|-----|-------|-----------------|-----------------|-----------------|
| 100 | $A_0$ | 1.00063318 E16  | 9.99800010 E15  | 9.99800010 E15  |
|     | $A_1$ | -4.00180197 E12 | -3.99926003 E12 | -3.99940002 E12 |
|     | $A_2$ | 6.00366045 E8   | 5.99941092 E8   | 5.99941093 E8   |
|     | $A_3$ | -4.00415037 E4  | -3.99994000 E4  | -3.99980000 E4  |
|     | $A_4$ | 1.00166458 E0   | 1.00000000 E0   | 1.00000000 E0   |
| 200 | $A_0$ | 2.56200523 E18  | 2.55987200 E18  | 2.55987200 E18  |
|     | $A_1$ | -2.56150866 E14 | -2.55988160 E14 | -2.55990440 E14 |
|     | $A_2$ | 9.60656082 E9   | 9.59976109 E9   | 9.59976109 E9   |
|     | $A_3$ | -1.60167819 E5  | -1.59999400 E5  | -1.59998000 E5  |
|     | $A_4$ | 1.00166458 E0   | 1.00000000 E0   | 1.00000000 E0   |
| 300 | $A_0$ | 6.56632158 E19  | 6.56085420 E19  | 6.56085420 E19  |
|     | $A_1$ | -2.91779343 E15 | -2.91594006 E15 | -2.91595140 E15 |
|     | $A_2$ | 4.86338852 E10  | 4.85994611 E10  | 4.85994611 E10  |
|     | $A_3$ | -3.60378345 E5  | -3.59999400 E5  | -3.59998000 E5  |
|     | $A_4$ | 1.00166458 E0   | 1.00000000 E0   | 1.00000000 E0   |

which this occurs is somewhat less for the complete equation than for the simplified equation.

It is to be re-emphasized that quite small differences in the coefficients of the characteristic equation lead to large differences in the roots. Some of the sets of coefficients are shown in Table 2. Such findings indicate, it would appear, that attempts to "improve" simplified versions of the theory of cylindrical shells may be ill-advised. The relative ease with which roots of the characteristic equation may be obtained by use of a digital computer also makes such attempts meaningless.

## References

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